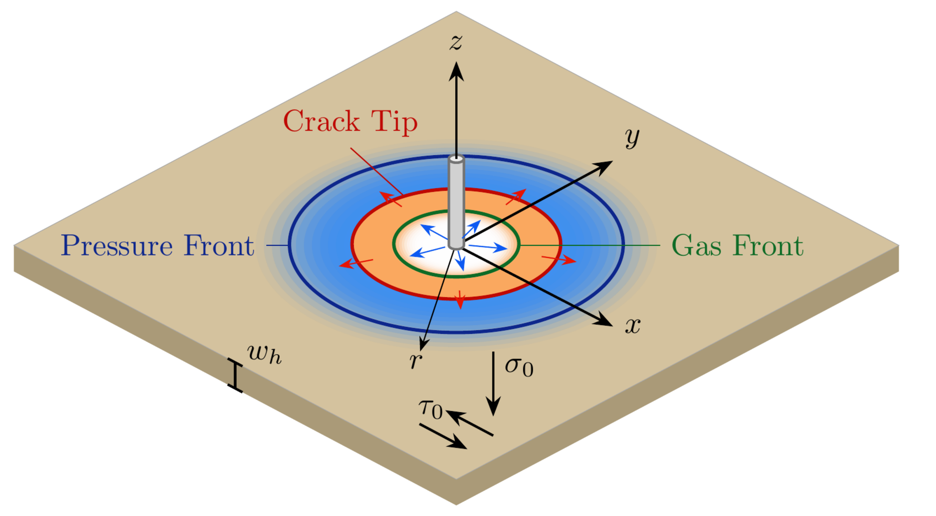
Gas compressibility suppresses fault rupture propagation.

Single-phase models provide a **conservative upper bound** on rupture size and seismic hazard for CO_2 storage.

An Analytical Coupling of Two-Phase Flow to Fault Slip

Seismicity at carbon capture and storage (CCS) sites makes it imperative to model fault slip under two-phase flow *(Vilarrasa et al., 2019)*. We present the first analytical model that relate rupture and pressure length scales for a self-similar formulation. We show that two-phase flow is self-similar and how the solutions scale under different regimes. We conducted a general parametric study using realistic supercritical CO_2 and fault parameters. We then apply the model to interpret experimentally induced slip at LSBB *(Guglielmi et al., 2015)*.



Self-similar pressure:
$p(r,t) = p_0 + \Delta p_* \, \Pi(\xi)$

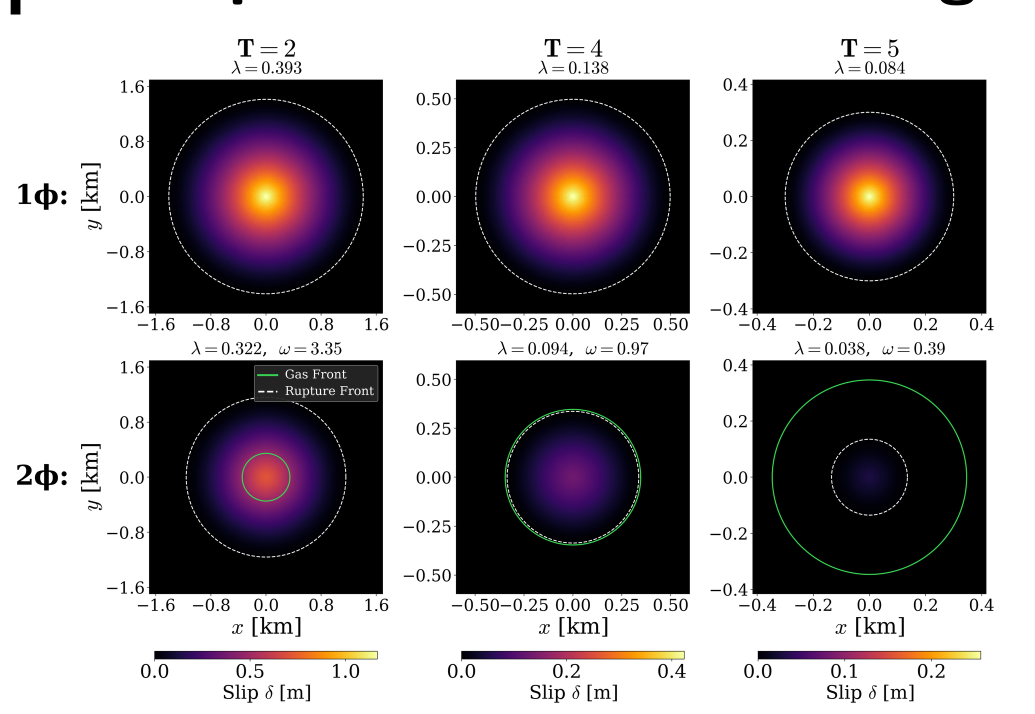
Coulomb failure:
$|\tau(x,y,t)| \le f\big(\sigma_0 - p(r,t)\big)$

Propagation condition:
$$\int_0^1 \frac{s\,\Pi(s^2\lambda^2)}{\sqrt{1-s^2}}\,ds = T$$

Stress-injection:
$$T := \frac{1-\tau_0/(f\sigma_0')}{\Delta p_*/\sigma_0'}$$

Building off the construction in *(Saez et al., 2022)*. Assume zero fracture toughness. T measures how far the fault is from failure against how pressurized it is.

2. Ruptures are smaller for gas injection



Representative fluid and fault parameters:

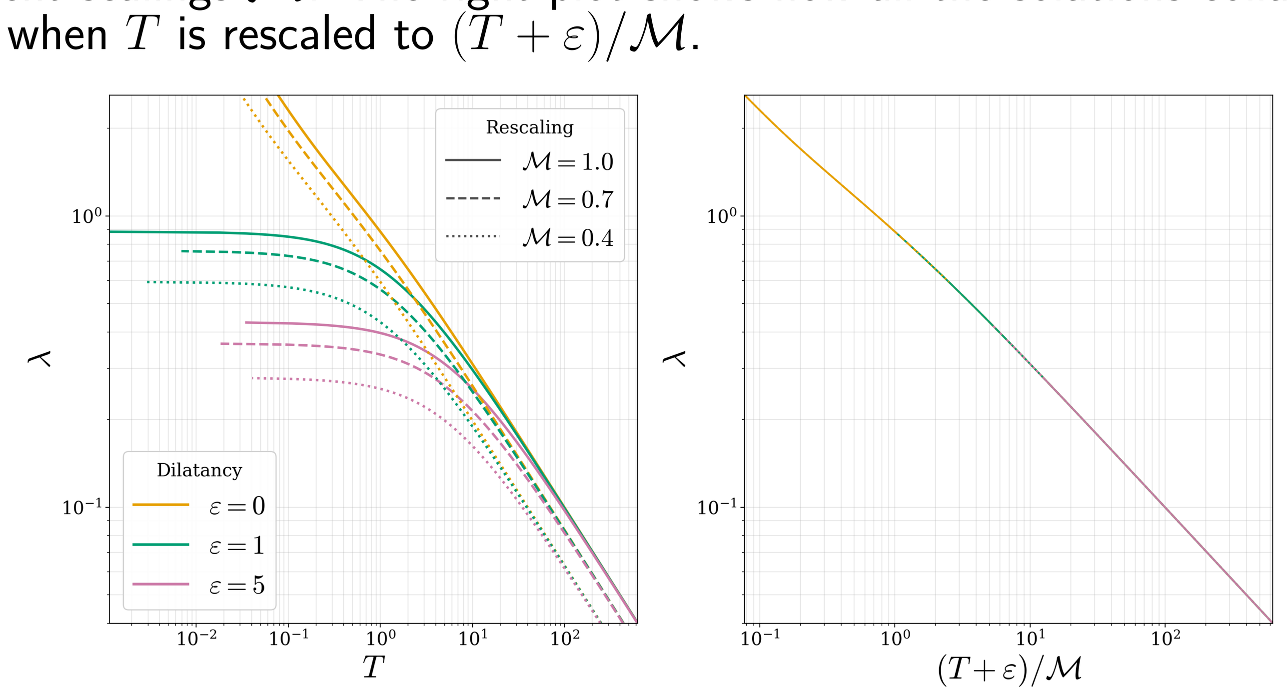
Parameter	Value	Physical meaning
p_0	21.65 MPa	Initial pressure at ~ 2 km
k	9.87×10^{-14} m^2	Fault permeability
φ_0	0.1	Porosity
w_h	1.88 m	Fault thickness
Q_m	1000 t/day	Injection rate
μ_{w0}	9.20×10^{-4} Pa·s	Constant water viscosity
μ_{g0}	7.18×10^{-5} Pa·s	Initial CO_2 viscosity
c_w	4.18×10^{-10} Pa^{-1}	Water compressibility
c_r	4.35×10^{-10} Pa^{-1}	Rock compressibility
$\alpha_{2\phi}$	1987.8 m^2/day	Two-phase diffusivity
$\alpha_{1\phi}$	1.09×10^{5} m^2/day	Single-phase diffusivity

Γ	0.2645
$M_0 = \mu_{g0}/\mu_{w0}$	0.0780
$\beta = (c_w + c_r)\,p_0$	0.0185

BEM-computed rupture footprints for three values of T. Top: single-phase (1ϕ). Bottom: two-phase (2ϕ). λ relates the rupture radius $R(t)$ to the diffusive length scale $L(t)$: $R(t) = \lambda\,L(t)$. $\lambda > 1$ means rupture outruns the pressure front while $\lambda < 1$ means it stays inside. ω relates the rupture radius $R(t)$ to gas front location r_g: $R(t) = \omega\,r_g(t)$. $L(t)$ not plotted so the rupture is visible. T is varied between columns to show how distance from failure affects the difference in rupture size for single and two phase flow at equivalent injection rates.

4. Dilatancy and permeability enhancement

Confining flow to the rupture makes the crack tip a moving boundary (eq. shown below): the water flux arriving at the tip supplies the plastic porosity increase $\Delta\varphi$ and elastic storage *(Dunham, 2024)*. We show the solution for two-phase flow under these conditions for when the crack is beyond the gas front. Dilatancy suppresses rupture growth in the critically stressed regime $T \ll 1$, with suppression amplified by two-phase effects through $\mathcal{M}$. The same scalar rescaling applies, now to the dilatancy parameter ε as well as to T. Left plot shows the impact of dilatancy and two-phase suppression with different scalings $\mathcal{M}$. The right plot shows how all the solutions collapse onto a single line when T is rescaled to $(T+\varepsilon)/\mathcal{M}$.



$$\text{BC:}\quad -\frac{k}{\mu_w}\frac{dp}{dr}\bigg|_{R^-} = \dot R\big(\Delta\varphi + c_t\varphi_0(p(R,t)-p_0)\big),\qquad \varepsilon := \frac{\Delta\varphi}{\varphi_0 c_t \Gamma p_0},\qquad \boxed{\lambda_{2\phi}^{\mathrm{conf}} \approx \lambda_{1\phi}^{\mathrm{conf}}\Big(\frac{T}{\mathcal{M}}, \frac{\varepsilon}{\mathcal{M}}\Big)}$$

References:

Guglielmi, Y., Cappa, F., Avouac, J.-P., Henry, P., Elsworth, D. (2015), Seismicity triggered by fluid injection-induced aseismic slip, *Science* **348**.

Vilarrasa, V., Carrera, J., Olivella, S., Rutqvist, J., Laloui, L. (2019), Induced seismicity in geologic carbon storage, *Solid Earth* **10**, 871–892.

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1. Radial two-phase flow is self-similar

Assumptions: immiscible, isothermal, radial flow into a homogeneous fault of constant permeability k; gravity and capillarity neglected. Injection at constant *mass* rate Q_m.

Mass balance for water (w) and gas (g):

$$\frac{\partial}{\partial t}\Big[\overbrace{\varphi(p)}^{\text{porosity}}\,\overbrace{\rho_w(p)}^{\text{density}}\,\overbrace{S_w}^{\text{saturation}}\Big] + \frac{1}{r}\frac{\partial}{\partial r}\big[r\,\rho_w(p)\,u_w(p,S_g)\big] = 0$$

$$\underbrace{\frac{\partial}{\partial t}\big[\varphi(p)\,\rho_g(p)\,S_g\big]}_{\text{storage}} + \underbrace{\frac{1}{r}\frac{\partial}{\partial r}\big[r\,\rho_g(p)\,u_g(p,S_g)\big]}_{\text{flux}} = \underbrace{\frac{Q_m}{2\pi r w_h}\delta(r)}_{\text{source}}$$

Darcy's law: $u_\zeta = -\big(k\,k_{r\zeta}(S_g)/\mu_\zeta(p)\big)\,\partial p/\partial r$, closure: $S_w + S_g = 1$

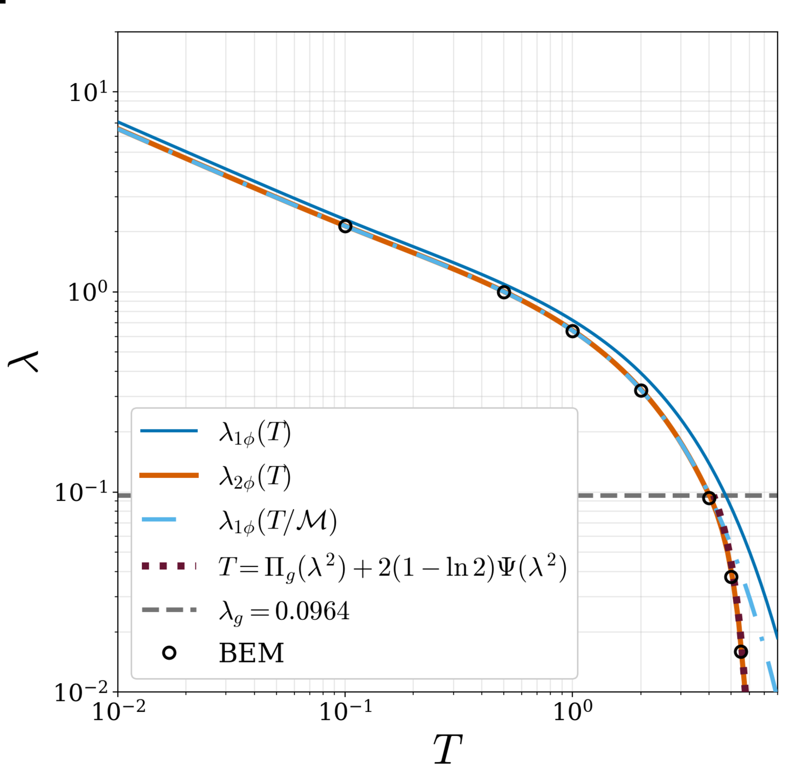
Substituting $\eta := r^2/(4\alpha_{2\phi}t)$ eliminates r and t:

$$\boxed{\begin{aligned} -\eta\frac{d}{d\eta}\big[\hat\varphi(\hat p)\,\hat\rho_w(\hat p)\,(1-S_g)\big] &= \frac{d}{d\eta}\left[\eta\,\frac{\hat\rho_w(\hat p)\,k_{rw}(S_g)}{\hat\mu_w(\hat p)}\frac{d\hat p}{d\eta}\right] \\ -\eta\frac{d}{d\eta}\big[\hat\varphi(\hat p)\,\hat\rho_g(\hat p)\,S_g\big] &= \frac{1}{M_0}\frac{d}{d\eta}\left[\eta\,\frac{k_{rg}(S_g)\,\hat\rho_g(\hat p)}{\hat\mu_g(\hat p)}\frac{d\hat p}{d\eta}\right] \end{aligned}}$$

Source BC: $\displaystyle \frac{1}{M_0}\lim_{\eta\to 0}\,\eta\,\frac{k_{rg}(S_g)\,\hat\rho_g(\hat p)}{\hat\mu_g(\hat p)}\frac{d\hat p}{d\eta} = \Gamma$, $\quad\alpha_{2\phi} := \dfrac{kp_0}{\varphi_0\mu_{w0}}$

Dimensionless injection rate: $\Gamma := \dfrac{Q_m\mu_{w0}}{4\pi w_h k p_0 \rho_{g0}}$, $\quad M_0 := \dfrac{\mu_{g0}}{\mu_{w0}}$, $\quad$ single-phase coordinate $\xi := c_t\,p_0\,\eta$

3. Gas compressibility suppresses rupture growth



For ruptures ahead of the gas front

Pressure beyond the gas front:

$\hat p(\xi) = 1 + A\,E_1(\xi), \quad A \in \mathbb{R}$

$\mathcal{M} := \frac{A}{\Gamma} \approx \frac{1}{\hat\rho_g(\hat p_f)} < 1$

$\Pi_{2\phi}(\xi) \approx \mathcal{M}\,\Pi_{1\phi}(\xi)$

$\Rightarrow \boxed{\lambda_{2\phi}(T) \approx \lambda_{1\phi}(T/\mathcal{M})}$

For ruptures inside the gas plume

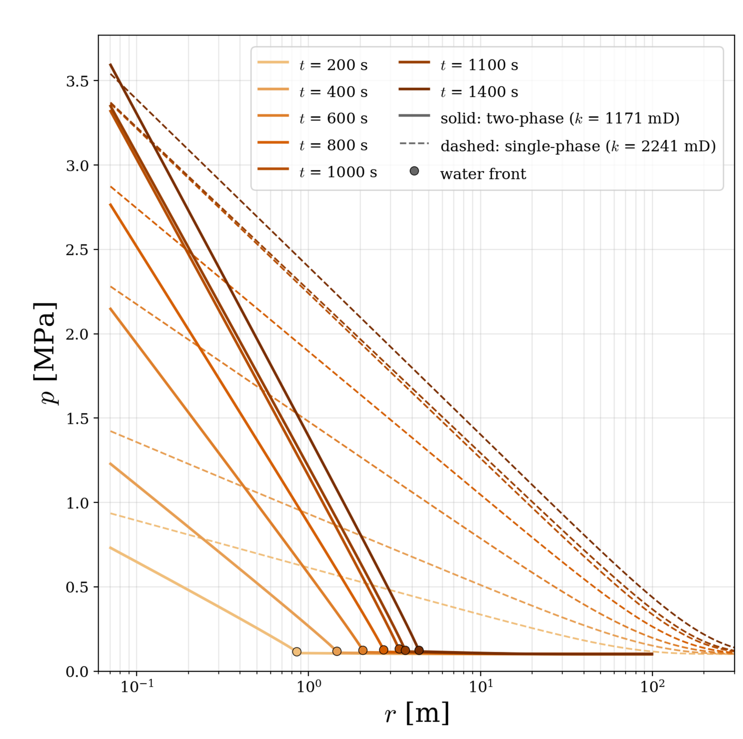
$$T \approx \Pi_g(\lambda^2) + \frac{2(1-\ln 2)}{\hat\rho_g\,\lambda_c}$$

λ, defined in Section 2, is a proxy for rupture size. $\lambda_{1\phi}(T)$ shows the curve relating λ and T for single-phase flow. $\lambda_{2\phi}$ shows the same curve for two-phase flow. $\lambda_{1\phi}(T/\mathcal{M})$ shows the asymptotic solution for when the crack is past the gas front, in the region where the pressure solution is the single-phase solution rescaled by $\mathcal{M}$. $\mathcal{M} < 1$ because gas compressibility absorbs injected mass before it pressurizes the water zone. $T = \Pi_{2\phi}(\lambda^2) + 2(1-\ln 2)/(\hat\rho_g\lambda_c)$ denotes the asymptotic solution for when the crack is behind the gas front. We Taylor expand about the crack tip since the crack propagation kernel decays exponentially away from the tip. λ_g denotes the location of the gas plume front. Circles show numerical validation points computed via BEM.

5. Application to water injection into a dry fault

Guglielmi et al. (2015) inject water into a dry fault at LSBB. We show the first two-phase interpretation of this experiment. We match the cumulative slip at the end of aseismic slip at 1100 seconds with two-phase and single-phase models. We set permeability by matching recorded well pressure, with $k_{1\phi} = 2.21\times10^{-12}$ m^2 and $k_{2\phi} = 1.16\times10^{-12}$ m^2 and. Initial water saturation of the two-phase model is set to 0.2 to account for potential residual water. The two-phase model requires a lower friction coefficient (0.54) than the single-phase model (0.85). The final rupture sizes for the two-phase and single-phase models are 19 m and 7 m, respectively. At matched friction the two-phase rupture size is smaller. The figure shows the evolution of pressure for both models. Notably, pressure decays more rapidly for the two-phase model. Gas compressibility, as for CO_2 injection into brine, suppresses pressure and rupture propagation in dry faults.

Parameter	Value
p_0	101.3 kPa (1 atm)
φ	0.1
w_h	0.2 m
μ_w	1.227×10^{-3} Pa·s
μ_g	1.78×10^{-5} Pa·s
c_w	4.50×10^{-10} Pa^{-1}
c_r	4.35×10^{-10} Pa^{-1}
$\alpha_{2\phi}$	10.6 m^2/s
$\alpha_{1\phi}$	20.4 m^2/s
G	10 GPa
τ_0	2.15 MPa
σ_0'	4.00 MPa



Dunham, E.M. (2024), Fluid-driven aseismic fault slip with permeability enhancement and dilatancy, *Phil. Trans. R. Soc. A* **382**.

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